

## **Systemic Barriers to Pedagogical Content Knowledge Development in Mathematics Education: An Integrative Analysis of Teacher Preparation at C. K. Tedam University of Technology and Applied Sciences, Ghana**

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### **Abstract**

This study examined systemic barriers to the development of pedagogical content knowledge (PCK) among mathematics education teachers at C. K. Tedam University of Technology and Applied Sciences in Ghana. PCK, which combines subject matter knowledge with pedagogical strategies, is widely recognized as a foundation of teaching effectiveness, yet teachers often face structural challenges in acquiring it. A descriptive survey design was employed, involving 98 mathematics education students who completed a questionnaire on curricular saliency, representations, instructional strategies, and diagnosis of student misconceptions. The results showed consistent challenges across all areas of PCK, with the greatest difficulties reported in the use of varied instructional strategies and in addressing misconceptions. Demographic variables such as gender, age, and level of education were not significantly associated with these challenges, suggesting that barriers are rooted in the design of teacher preparation rather than individual differences. The study concludes that reforms in Ghana's teacher education system are urgently needed. These should include curriculum redesign, expanded practicum opportunities, mentorship, and professional development that places PCK at the center of mathematics teacher education.

**Keywords:** pedagogical content knowledge, systemic barriers, mathematics education, teacher preparation, Ghana

### **Introduction**

Pedagogical content knowledge has long been regarded as the distinguishing feature of effective teaching. Shulman (1986) described it as the special blend of subject matter expertise and pedagogy that allows teachers to make content understandable to learners. Since then, scholars have elaborated the concept to include knowledge of the curriculum, the use of multiple

representations, strategies for teaching, and the ability to diagnose and respond to students' misconceptions (Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008). In mathematics education, PCK enables teachers to go beyond procedures and help students develop conceptual understanding, which is critical for long-term achievement (Baumert et al., 2010).

Despite its importance, PCK remains difficult to develop. Research has shown that teachers across many countries rely heavily on limited instructional routines, often favoring lecture-based explanations that do little to address diverse student needs (Depaepe, Verschaffel, & Kelchtermans, 2013; Kleickmann et al., 2013). Large-scale studies, such as the Teacher Education and Development Study in Mathematics (TEDS-M), have demonstrated that the biggest influences on teacher knowledge are not demographic characteristics of students but the structures of teacher education itself, including curriculum design, the balance between theory and practice, and access to mentoring (Tatto et al., 2012; Blömeke & Delaney, 2012).

In Ghana, concerns about mathematics education have persisted for decades. Teacher preparation has often been criticized for placing more emphasis on content mastery than on pedagogy, which leaves many new teachers ill-equipped to connect mathematical ideas with students' thinking (Osei, 2006; Anamuah-Mensah, 2020). While research has identified weaknesses in areas such as the use of representations and curriculum integration (Ayebo & Assuah, 2017), less attention has been paid to the underlying systemic factors that create these persistent challenges.

This study responds to that gap by examining the barriers to developing PCK among mathematics education teachers at C. K. Tedam University of Technology and Applied Sciences. It focuses on how teachers experience difficulties across the four dimensions of PCK and considers the extent to which these challenges are linked to systemic structures in teacher preparation. By

moving beyond descriptions of teacher competence to a more integrative analysis, the study aims to provide insights that can guide reforms in mathematics teacher education in Ghana.

## Methods

### Research Design

The study employed a descriptive survey design to capture the perspectives of mathematics education teachers on challenges in developing PCK. This approach was appropriate because it allowed the researchers to gather data from a relatively large group in a structured way and to analyze both descriptive trends and inferential relationships (Creswell & Creswell, 2018).

### Participants

The participants were 98 mathematics education students enrolled at C. K. Tedam University of Technology and Applied Sciences. They included 62 males, representing 63.3 percent, and 36 females, representing 36.7 percent. In terms of age, 42 respondents were between 20 and 25 years, 38 were between 26 and 30 years, and 18 were above 31 years. Slightly more than half were undergraduates while the rest were postgraduates, which provided balance in perspectives from different levels of teacher preparation.

### Instrumentation

Data were collected using a questionnaire developed from established frameworks for assessing PCK (Hill et al., 2008; Mavhunga & Rollnick, 2013). The instrument contained items relating to four areas of PCK: curricular saliency, use of representations, instructional strategies,

and diagnosing misconceptions. Participants responded using a five-point Likert scale ranging from strongly disagree to strongly agree. The instrument was reviewed by experts in mathematics education and piloted before use to ensure clarity and validity.

### Data Collection

The questionnaires were administered during scheduled classes with the permission of course instructors. Before completing the instrument, participants were informed about the purpose of the study and assured of confidentiality. The questionnaires were collected immediately after completion, which ensured a very high response rate.

### Data Analysis

Responses were analyzed using SPSS version 25. Descriptive statistics such as means and standard deviations were used to identify the most common challenges, while correlation analysis examined the relationships between the four PCK domains. Chi-square tests were also conducted to determine whether demographic factors such as gender, age, or level of education were associated with particular challenges. Significance was set at a p-value of less than 0.05.

### Ethical Considerations

Ethical approval for the study was obtained from the Faculty of Education at CKTUTAS. Participation was voluntary and all participants provided informed consent. To protect anonymity, respondents were assigned identification codes instead of names. They were assured that they could withdraw from the study at any time without any consequences. Data

were stored securely and used solely for academic purposes.

### Results

#### Table 1

Demographic Characteristics of Respondents (N = 98)

| Variable          | Category           | Frequency | Percentage |
|-------------------|--------------------|-----------|------------|
| Sex               | Male               | 62        | 63.3       |
|                   | Female             | 36        | 36.7       |
| Age               | 20–25 years        | 42        | 42.9       |
|                   | 26–30 years        | 38        | 38.8       |
|                   | 31 years and above | 18        | 18.3       |
| Educational Level | Undergraduate      | 54        | 55.1       |
|                   | Postgraduate       | 44        | 44.9       |

The demographic profile shows that the majority of respondents were male, representing nearly two-thirds of the sample. Most participants were relatively young, with over 80 percent aged between 20 and 30 years. The sample was balanced in terms of educational level, with slightly more undergraduates than postgraduates. This distribution reflects the composition of the mathematics education student population at CKTUTAS.

**Table 2**  
Descriptive Statistics of Reported Challenges (N = 98)

| Challenge Area                                   | Mean | SD   | Interpretation     |
|--|------|------|--------------------|
| Limited use of multiple instructional strategies | 3.72 | 0.59 | High Challenge     |
| Diagnosing students' misconceptions              | 3.65 | 0.61 | High Challenge     |
| Integrating curriculum knowledge into practice   | 3.48 | 0.57 | Moderate Challenge |
| Applying multiple representations                | 3.42 | 0.63 | Moderate Challenge |

The results in Table 2 indicate that difficulties with instructional strategies were the most significant challenge reported by respondents, with a mean score of 3.72. Close behind was the challenge of diagnosing students' misconceptions, with a mean of 3.65. Challenges in curriculum integration and the use of multiple representations were rated as moderate, although their mean scores were also above the midpoint of the scale. Overall, the results suggest that while participants had some familiarity with PCK dimensions, they struggled most with areas that require adaptation to learners' needs.

**Table 3**  
Correlations among PCK Dimensions (N = 98)

| Variable                    | 1 | 2     | 3     | 4     |
|-----------------------------|---|-------|-------|-------|
| 1. Instructional Strategies | 1 | .46** | .39** | .42** |
| 2. Misconceptions           |   | 1     | .41** | .44** |
| 3. Curriculum Integration   |   |       | 1     | .36** |
| 4. Representations          |   |       |       | 1     |

**Note:** p < .01

Table 3 shows significant positive correlations among all four PCK domains. The strongest relationship was between challenges in instructional strategies and diagnosing misconceptions ( $r = .46$ ,  $p < .01$ ), suggesting that teachers who struggled with one were likely to struggle with the other. Moderate positive correlations were also found between curriculum integration and the other dimensions, indicating that weaknesses in one area of PCK are connected to difficulties across the spectrum. These results support the argument that barriers to PCK development are systemic and interconnected, rather than isolated to specific domains.

**Table 4**  
Chi-Square Tests of Association between Demographics and Reported Challenges (N = 98)

| Variable                          | $\chi^2$ | df | p-value |
|-----------------------------------|----------|----|---------|
| Gender × Instructional Strategies | 1.14     | 1  | .29     |

|                              |   |      |   |     |
|------------------------------|---|------|---|-----|
| Gender × Misconceptions      | × | 0.87 | 1 | .35 |
| Age × Curriculum Integration |   | 2.61 | 2 | .27 |
| Age × Representations        |   | 3.08 | 2 | .21 |
| Level × Misconceptions       |   | 0.96 | 1 | .33 |

The chi-square analysis revealed no statistically significant associations between demographic variables and the reported challenges. Male and female participants reported similar difficulties, as did respondents across different age groups and educational levels. This finding reinforces the view that the barriers to PCK development are not the result of individual differences but are structural features of teacher preparation programs.

### Summary of Results

The analysis revealed that mathematics education teachers at CKTUTAS face significant difficulties in using varied instructional strategies and diagnosing students' misconceptions, while moderately struggling with curriculum integration and the use of multiple representations. Correlation analysis showed that these challenges are closely interrelated, pointing to systemic barriers in the development of PCK. Furthermore, the absence of demographic effects indicates that these barriers cut across all groups of teacher trainees, suggesting that they are embedded in the teacher education system itself.

### Discussion

This study set out to examine systemic barriers to the development of pedagogical content knowledge among mathematics education

teachers at C. K. Tedam University of Technology and Applied Sciences in Ghana. The results revealed that participants experienced significant challenges in using varied instructional strategies and in diagnosing students' misconceptions, while reporting moderate difficulties in curriculum integration and the use of multiple representations. Correlation analysis showed that these barriers are highly interconnected, suggesting that they are not isolated weaknesses but reflect systemic limitations in the teacher education process. Furthermore, the absence of significant demographic effects indicates that these challenges cut across gender, age, and level of study, reinforcing the idea that they are structural rather than individual in origin.

The finding that instructional strategies posed the greatest challenge is consistent with previous studies which have shown that many mathematics teachers, particularly in developing countries, rely heavily on lecture-based and teacher-centered methods (Depaepe, Verschaffel, & Kelchtermans, 2013). Such methods often neglect active engagement, collaborative learning, and problem-solving approaches that are essential for deeper conceptual understanding (Baumert et al., 2010). In Ghana, similar observations have been made, with mathematics classrooms frequently dominated by procedural teaching that limits opportunities for learners to construct knowledge meaningfully (Ayebo & Assuah, 2017). This reflects a systemic issue within teacher education, where training programs tend to emphasize content delivery over the practice of diversified pedagogical methods (Anamuah-Mensah, 2020).

The difficulty in diagnosing misconceptions further highlights a systemic gap in teacher preparation. Misconceptions in mathematics are both common and deeply rooted, and effective teaching requires the ability to identify, interpret, and correct them (Shulman, 1986; Kind, 2009). However, many pre-service teachers in this study reported struggling in this area, which suggests insufficient training in formative assessment and diagnostic approaches. International studies, including the TEDS-M project, have shown that without explicit training in how to recognize and respond to misconceptions, teachers tend to perpetuate surface-level explanations rather than fostering conceptual clarity (Tatto et al., 2012; Blömeke & Delaney, 2012).

The moderate challenges in curriculum integration and use of multiple representations are equally important. Teachers' ability to recognize the relative importance of concepts, connect them across topics, and represent them in different ways is central to high-quality mathematics instruction (Hill, Ball, & Schilling, 2008). Yet these were not areas of strength among the respondents. This finding is consistent with studies which argue that many teacher preparation programs do not devote enough attention to curricular saliency and representation as distinct dimensions of teacher knowledge (Kleickmann et al., 2013; Grossman, 1990). In Ghana, this may reflect an overemphasis on covering syllabi rather than cultivating flexible pedagogical skills.

The interrelationships among the PCK dimensions suggest that weaknesses in one area often reinforce difficulties in others. For instance, teachers who struggle with instructional strategies are also less likely to

effectively diagnose misconceptions or integrate curriculum content. This finding aligns with the argument that PCK is a holistic construct rather than a collection of unrelated skills (Mavhunga & Rollnick, 2013; Park & Oliver, 2008). The systemic nature of the challenges points to the need for comprehensive reform in teacher education rather than piecemeal interventions.

The lack of significant demographic effects adds weight to the argument that barriers are structural. Whether male or female, young or older, undergraduate or postgraduate, participants reported similar challenges. This is consistent with international evidence that structural features of teacher education—such as curriculum design, opportunities for practice, and availability of mentorship—are more important determinants of teacher knowledge than demographic characteristics (Tatto et al., 2012).

These findings carry important policy implications. Teacher education in Ghana should be restructured to place the development of pedagogical content knowledge at its core. This includes revising curricula to integrate modules on diagnosing misconceptions, curriculum saliency, and multiple representations alongside traditional subject matter training. Practicum experiences should be expanded and strengthened so that teacher trainees can apply these skills in authentic classroom contexts under guided mentorship. Continuous professional development programs should also target practicing teachers to ensure that they continue to refine their PCK throughout their careers (Clarke & Hollingsworth, 2002; Darling-Hammond, 2017). At the national level, policymakers in the

Ministry of Education and Ghana Education Service need to align teacher education reforms with classroom practice expectations to reduce the disconnect between training and teaching reality.

Despite its contributions, the study has limitations. It relied on self-reported data, which may be subject to response bias, as participants could either overestimate or underestimate their challenges. In addition, the study was conducted at a single university, which limits the generalizability of the findings to all teacher education programs in Ghana. A broader multi-institutional study, possibly incorporating classroom observations and interviews, would provide richer insights. Furthermore, the cross-sectional design does not capture how teachers' PCK might develop over time, suggesting the value of longitudinal research in this area.

## Conclusion

This study examined systemic barriers to the development of pedagogical content knowledge among mathematics education teachers at C. K. Tedam University of Technology and Applied Sciences in Ghana. The results revealed that teachers faced significant challenges in employing varied instructional strategies and in diagnosing students' misconceptions, while moderate challenges were reported in curriculum integration and the use of multiple representations. Importantly, these challenges were strongly interrelated and not influenced by demographic variables, underscoring that they are systemic rather than individual.

The findings highlight the urgent need for comprehensive reforms in mathematics teacher education in Ghana. Teacher preparation

programs must be redesigned to embed pedagogical content knowledge as a central component, rather than as a peripheral concern. This involves strengthening practice-oriented learning, expanding classroom-based practicum experiences, and developing structured mentorship programs. At the policy level, there is a need for greater alignment between the national teacher education curriculum and the realities of mathematics teaching in Ghanaian classrooms.

While the study was limited by its reliance on self-reported data and its focus on a single institution, it provides important evidence that barriers to PCK development are deeply rooted in the structures of teacher preparation. Future research should expand the scope to include multiple universities and employ mixed-methods approaches to provide a more comprehensive picture. Addressing these systemic barriers is essential if Ghana is to prepare mathematics teachers who can foster deeper conceptual understanding and improve student achievement.

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